# Nekonečná geometrická řada

**Příklady**

① Dána geometrická posloupnost $(a_n)_{n=1}^{\infty}$, $a_n = \left(\frac{1}{2}\right)^n$:

a) Určete prvních 6 členů, kvocient a limitu

$$a_n = \left(\frac{1}{2}\right)^n = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}$$

GP: $a_n = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1} \Rightarrow a_1 = \frac{1}{2},\ q = \frac{1}{2}$

$a_n = a_1 q^{n-1}$

$\underbrace{|q| < 1}_{\text{podmínka konvergence}} \Rightarrow \lim\limits_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$

členy: $a_1 = \frac{1}{2}$ $a_2 = \frac{1}{4}$ $a_3 = \frac{1}{8}$ $a_4 = \frac{1}{16}$ $a_5 = \frac{1}{32}$ $a_6 = \frac{1}{64}$

b) vytvořte novou posloupnost $(s_n)_{n=1}^{\infty}$, kde $s_n = a_1 + a_2 + \ldots + a_n$ a zjistěte, zda je konvergentní.

$s_1 = a_1 = \frac{1}{2}$

$s_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$s_3 = a_1 + a_2 + a_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

$s_4 = a_1 + a_2 + a_3 + a_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$

POSLOUPNOST ČÁSTEČNÝCH SOUČTŮ $(s_n)_{n=1}^{\infty}$

⋮

$$s_n = \underbrace{a_1\frac{q^n - 1}{q - 1}}_{\text{součet } n \text{ členů GP}} = a_1 + a_2 + \ldots + a_n = \frac{1}{2}\,\frac{\left(\frac{1}{2}\right)^n - 1}{\frac{1}{2} - 1} = \frac{1}{2}\,\frac{\left(\frac{1}{2}\right)^n - 1}{-\frac{1}{2}} = 1 - \left(\frac{1}{2}\right)^n$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}\left(1 - \left(\tfrac{1}{2}\right)^n\right) = \lim_{n\to\infty} 1 - \lim_{n\to\infty}\left(\tfrac{1}{2}\right)^n = 1 - 0 = 1$$

**DEFINICE**

Nechť je dána posloupnost $(a_n)_{n=1}^{\infty}$.
NEKONEČNOU ŘADOU nazýváme výraz $a_1 + a_2 + a_3 + \ldots + a_n + \ldots = \sum\limits_{n=1}^{\infty} a_n$
ČLENY nekonečné řady jsou členy dané posloupnosti.

**VĚTA**

NEKONEČNÁ ŘADA je KONVERGENTNÍ $\Leftrightarrow$ posloupnost částečných součtů $(s_n)_{n=1}^{\infty}$, kde $s_n = a_1 + a_2 + \ldots + a_n$, JE KONVERGENTNÍ
a SOUČET TÉTO ŘADY

$$s = \lim_{n\to\infty} s_n = \sum_{n=1}^{\infty} a_n$$

**DEFINICE**

Je-li $(a_n)_{n=1}^{\infty}$ GEOMETRICKÁ POSLOUPNOST s kvocientem $q$ $(a_1 \neq 0)$, pak příslušnou nekonečnou řadu

$$\sum_{n=1}^{\infty} a_n = a_1 + a_1 q + a_1 q^2 + \ldots + a_1 q^{n-1} + \ldots$$

nazýváme NEKONEČNÁ GEOMETRICKÁ ŘADA

VĚTA

je-li $(a_n)_{n=1}^{\infty}$ GEOMETRICKÁ POSLOUPNOST s kvócientem $q$, pak NEKONEČNÁ GEOMETRICKÁ ŘADA je pro $|q|<1$ $(a_1 \neq 0)$ KONVERGENTNÍ a její součet je $s = \sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \frac{a_1}{1-q}$

[pro $|q| \geq 1$ je DIVERGENTNÍ]

[důkaz:

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} a_1 \frac{q^n-1}{q-1} = \lim_{n\to\infty} \underbrace{\frac{a_1}{q-1}}_{C\in R} \cdot \lim_{n\to\infty}(q^n-1) =$$

$$= \lim_{n\to\infty} \underbrace{\frac{a_1}{q-1}}_{C\in R} \big(\underbrace{\lim_{n\to\infty} q^n}_{=0 \text{ pro } |q|<1} - \lim_{n\to\infty} 1\big) = \frac{a_1}{q-1}(0-1) = -\frac{a_1}{q-1} = \frac{a_1}{1-q} \text{ cbd.}$$
]

př. $\left(\left(-\frac{1}{3}\right)^{n-1}\right)_{n=1}^{\infty}$ $\quad a_n = \left(-\frac{1}{3}\right)^{n-1} = 1\cdot\left(-\frac{1}{3}\right)^{n-1}$ $\quad a_1 = 1$ $\quad a_2 = \left(-\frac{1}{3}\right)^{2-1} = -\frac{1}{3}$ $\quad a_3 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$ $\quad a_4 = \left(-\frac{1}{3}\right)^3 = -\frac{1}{27}$

$a_n = a_1 q^{n-1}$

GP: $a_1 = 1$, $q = -\frac{1}{3}$

$|q| < 1$ PODMÍNKA KONVERGENCE PLATÍ

$\Rightarrow$ KONVERGUJE I NEKONEČNÁ GEOMETRICKÁ ŘADA a její SOUČET je $s = \lim_{n\to\infty} s_n = \frac{a_1}{1-q}$

$$s = \sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^{n-1} = 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots = \frac{1}{1-\left(-\frac{1}{3}\right)} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

př. $(2n)_{n=1}^{\infty}$

$\sum_{n=1}^{\infty} 2n = 2+4+6+8+\ldots$ DIVERGENTNÍ

Příklady

② Daná geom. posloupnost $((-1)^n)_{n=1}^{\infty}$. Urči několik členů posloupnosti $(s_n)_{n=1}^{\infty}$, kde $s_n = a_1 + a_2 + \ldots + a_n$ a dále zjisti, zda je dalo posloupnost částečných součtů konvergentní.

GP: $a_1 = -1$ $\quad a_2 = 1$ $\quad a_3 = -1$ $\quad a_4 = 1$ $\qquad q = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = -1$

podmínka konvergence $|q|<1$ NEPLATÍ $\Rightarrow ((-1)^n)_{n=1}^{\infty}$ DIVERG.

$\Rightarrow (s_n)_{n=1}^{\infty}$ je také divergentní

$(s_n)_{n=1}^{\infty}$

$$\left.\begin{array}{l} s_1 = a_1 = -1 \\ s_2 = a_1 + a_2 = -1+1 = 0 \\ s_3 = a_1+a_2+a_3 = -1+1-1 = -1 \\ s_4 = a_1+a_2+a_3+a_4 = -1+1-1+1 = 0 \\ \vdots \end{array}\right\} \Rightarrow \left.\begin{array}{l} s_{2k-1} = -1 \\ s_{2k} = 0 \end{array}\right\} \text{posloupnost částečných součtů } (s_n)_{n=1}^{\infty} \text{ je DIVERGENTNÍ}$$

③ Zjistěte, zda je nekonečná řada konvergentní a určete její součet

a) $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}} = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \ldots = \underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots}_{GŘ}$

$[1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ GP: $a_1 = 1$ $q = \frac{1}{2} \Rightarrow |q| < 1$ konv.$]$

GŘ: $a_1 = 1$, $q = \frac{1}{2}$

$|q| < 1$ podm. konv. platí $\Rightarrow$

$\Rightarrow s = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

b) $\sum_{n=1}^{\infty} 10^{-n} = 10^{-1} + 10^{-2} + 10^{-3} = \underbrace{\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3}}_{} + \ldots = \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$

GŘ: $q = \frac{a_2}{a_1} = \frac{\frac{1}{100}}{\frac{1}{10}} = \frac{10}{100} = \frac{1}{10}$

$a_1 = \frac{1}{10}$   $|q| < 1$ podmínka konvergence GŘ platí $\Rightarrow$

$\Rightarrow s = \frac{a_1}{1-q} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{10}{90} = \frac{1}{9}$

tedy: $s = 0{,}1 + 0{,}01 + 0{,}001 + \ldots = 0{,}1111\ldots = 0{,}\overline{1}$

$\Big\} \Rightarrow 0{,}\overline{1} = \frac{1}{9}$

c) DÚ $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{3}\right)^n = \sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n = \underbrace{\left(-\frac{2}{3}\right)^1 + \left(-\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^3}_{} + \ldots$

GŘ: $q = -\frac{2}{3}$   $a_1 = -\frac{2}{3}$

$|q| < 1$ platí podm. konv.

$\Rightarrow s = \frac{a_1}{1-q} = \frac{-\frac{2}{3}}{1-\left(-\frac{2}{3}\right)} = \frac{-\frac{2}{3}}{\frac{5}{3}} = -\frac{6}{15} = -\frac{2}{5}$

d) $\sum_{n=1}^{\infty} \frac{\sqrt{5}}{2^{n-1}} = \frac{\sqrt{5}}{2^0} + \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2^2} + \frac{\sqrt{5}}{2^3} + \ldots = \sqrt{5} + \sqrt{5} \cdot 2^{-1} + \sqrt{5} \cdot 2^{-2} + \sqrt{5} \cdot 2^{-3} + \ldots$

1. zp. GŘ: $q = \frac{\frac{\sqrt{5}}{2}}{\sqrt{5}} = \frac{\frac{\sqrt{5}}{2^2}}{\frac{\sqrt{5}}{2}} = \frac{1}{2}$   podm. konv. $|q| < 1$ platí

$\Rightarrow s = \lim_{n\to\infty} s_n = \frac{a_1}{1-q} = \frac{\sqrt{5}}{1-\frac{1}{2}} = \frac{\sqrt{5}}{\frac{1}{2}} = 2\sqrt{5}$

2. zp. upravme

$\sum_{n=1}^{\infty} \frac{\sqrt{5}}{2^{n-1}} = \sqrt{5} + \sqrt{5} \cdot 2^{-1} + \sqrt{5} \cdot 2^{-2} + \ldots$

$= \sqrt{5}\left(1 + 2^{-1} + 2^{-2} + \ldots\right) = \sqrt{5}\left(\underbrace{\left(\tfrac{1}{2}\right)^0 + \left(\tfrac{1}{2}\right)^1 + \left(\tfrac{1}{2}\right)^2 + \ldots}_{} \right)$

GŘ: $a_1 = 1 \left[\left(\frac{1}{2}\right)^0\right]$

$q = \frac{1}{2}$   $|q| < 1$ pl.

$\Rightarrow s = \sqrt{5}\, s' = \sqrt{5} \cdot 2 = 2\sqrt{5}$

$s' = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

④ napište ve tvaru zlomku s celočíselným čitatelem a jmenovatelem

a) $0,\overline{4} = 0,4444\ldots = 0,4 + 0,04 + 0,004 + 0,0004 + \ldots =$

1. způs. $= \underbrace{4\cdot10^{-1} + 4\cdot10^{-2} + 4\cdot10^{-3} + 4\cdot10^{-4}}_{GŘ} + \ldots$

$q = \frac{4\cdot10^{-2}}{4\cdot10^{-1}} = 10^{-1} = \frac{1}{10}$ $\qquad s = \frac{a_1}{1-q} = \frac{\frac{4}{10}}{1-\frac{1}{10}} = \frac{\frac{4}{10}}{\frac{9}{10}} = \frac{40}{90} = \frac{4}{9}$

$a_1 = 4\cdot10^{-1}$ $\qquad |q|<1$ platí $\qquad 0,\overline{4} = \frac{4}{9}$

lze i s využ. př. ③ b)

2. způs. $0,\overline{4} = 0,4444\ldots = 0,4 + 0,04 + 0,004 + 0,0004 + \ldots =$

$= 4(\underbrace{0,1 + 0,01 + 0,001 + 0,0001 + \ldots}_{GŘ}) =$

$q = \frac{0,01}{0,1} = 0,1 = \frac{1}{10}$

$0,\overline{4} = 4s = 4\cdot\frac{1}{9}$ $\qquad a_1 = 0,1 \qquad s = \frac{a_1}{1-q} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{9}$

$0,\overline{4} = \frac{4}{9}$

b) $-0,3\overline{25} = -(0,3 + 0,025 + 0,00025 + 0,0000025 + \ldots)$

$= -\left(\frac{3}{10} + \frac{25}{1000} + 25\cdot10^{-5} + 25\cdot10^{-7} + \ldots\right)$

$= -\left[\frac{3}{10} + 25(\underbrace{10^{-3} + 10^{-5} + 10^{-7} + \ldots}) \right] = -\left(\frac{3}{10} + 25\cdot\frac{1}{990}\right) =$

[GŘ: $q = 10^{-2}$ $\quad |q|<1$ podm. konv. platí

$s = \frac{a_1}{1-q} = \frac{10^{-3}}{1-10^{-2}} = \frac{\frac{1}{1000}}{1-\frac{1}{100}} = \frac{\frac{1}{1000}}{\frac{99}{100}} = \frac{100}{99\cdot1000} = \frac{1}{990}$ ]

$= -\frac{3\cdot99+25}{990} = -\frac{297+25}{990} = -\frac{322}{990} = -\frac{161}{495}$

[v 1. roč. ČTYŘ. (5. roč. OSMIL)

$0,3\overline{25} = 0,32525\ldots = \frac{3}{10} + 0,02525\ldots$

$a = 0,025|25\ldots$
$1000a = 25,2525\ldots$
$10a = 0,2525\ldots$ $\}\ominus$
$990a = 25$
$a = \frac{25}{990}$ ]

c) $-2,\overline{5} = -(2 + 0,5 + 0,05 + 0,005 + \ldots) = -2 - (\underbrace{5\cdot10^{-1} + 5\cdot10^{-2} + 5\cdot10^{-3}}_{GŘ} + \ldots) = -2 - s$

$-2,\overline{5} = -2 - \frac{5}{9} = -\frac{23}{9}$

$q = 10^{-1}$, $a_1 = 5\cdot10^{-1}$
$|q|<1$ pl.
$s = \frac{a_1}{1-q} = \frac{5\cdot10^{-1}}{1-\frac{1}{10}} = \frac{\frac{5}{10}}{\frac{9}{10}} = \frac{50}{90} = \frac{5}{9}$

d) $-0,8\overline{4} = -(0,8 + 0,04 + 0,004 + 0,0004 + \ldots) = -(0,8 + \underbrace{4\cdot10^{-2} + 4\cdot10^{-3}}_{GŘ} + \ldots) = -(0,8 + s)$

DÚ

$-0,8\overline{4} = -\left(\frac{8}{10} + \frac{4}{90}\right) = -\frac{72+4}{90} = -\frac{76}{90}$

$-0,8\overline{4} = -\frac{38}{45}$

$q = 10^{-1} = \frac{1}{10}$ $\qquad a_1 = 4\cdot10^{-2} = \frac{4}{100}$
$|q|<1$ pl.
$s = \frac{a_1}{1-q} = \frac{\frac{4}{100}}{1-\frac{1}{10}} = \frac{\frac{4}{100}}{\frac{9}{10}} = \frac{40}{900} = \frac{4}{90}$

e) $5,4\overline{87} = 5 + \frac{4}{10} + 87\cdot 10^{-3} + 87\cdot 10^{-5} + \dots =$

$$= \frac{54}{10} + 87(\underbrace{10^{-3} + 10^{-5} + \dots}) = \frac{54}{10} + \frac{87\cdot 1}{990} = \frac{54\cdot 99 + 87}{990} = \frac{5433}{990} = \frac{1811}{330}$$

[ GŘ: $q = 10^{-2}$ $|q| < 1$ podm. konv. platí

$s = \frac{a_1}{1-q} = \frac{10^{-3}}{1-10^{-2}} = \frac{\frac{1}{1000}}{1-\frac{1}{100}} = \frac{\frac{1}{1000}}{\frac{99}{100}} = \frac{100}{99\cdot 1000} = \frac{1}{990}$ ]

nebo

$$5,4\overline{87} = \frac{54}{10} + \underbrace{87\cdot 10^{-3} + 87\cdot 10^{-5} + 87\cdot 10^{-7} + \dots} = \frac{54}{10} + \frac{29}{33} = \frac{54\cdot 33 + 290}{330} = \frac{1811}{330}$$

[ GŘ: $q = 10^{-2}$ $|q| < 1$ podm. konv. platí

$s = \frac{a_1}{1-q} = \frac{87\cdot 10^{-3}}{1-10^{-2}} = \frac{\frac{87}{1000}}{1-\frac{1}{100}} = \frac{\frac{87}{1000}}{\frac{99}{100}} = \frac{87\cdot 100}{99\cdot 1000} = \frac{87}{99} = \frac{29}{33}$ ]

5) Vypočítejte hodnotu

a) $y = x\cdot\sqrt{x^3}\cdot\sqrt[4]{x^3}\cdot\sqrt[8]{x^3}\cdot\sqrt[16]{x^3}\dots$ $\quad D = R$

$y = x^1\cdot x^{\frac{3}{2}}\cdot x^{\frac{3}{4}}\cdot x^{\frac{3}{8}}\cdot x^{\frac{3}{16}}\cdot\dots$

$y = x^{1+\frac{3}{2}+\frac{3}{4}+\frac{3}{8}+\frac{3}{16}+\dots}$

$y = x^{1+3(\underbrace{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots}_{GŘ})} = x^{1+3s} = x^{1+3\cdot 1} = x^4$

[ $a_1 = \frac{1}{2}$ $q = \frac{1}{2}$ $|q| < 1$ pl.

$s = \frac{a_1}{1-q} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$ ]

b) $y = 3\sqrt{3}\cdot\sqrt[4]{3}\cdot\sqrt[8]{3}\cdot\sqrt[16]{3}\dots$

$y = 3\cdot 3^{\frac{1}{2}}\cdot 3^{\frac{1}{4}}\cdot 3^{\frac{1}{8}}\cdot 3^{\frac{1}{16}} = 3^{1+\underbrace{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots}_{GŘ}} = 3^{1+s} = 3^2 = 9$

[ $a_1 = \frac{1}{2}$ $q = \frac{1}{2}$ $|q| < 1$ pl.

$s = \frac{a_1}{1-q} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$ ]

c) $y = \frac{1+2+3+4+5+\dots+n}{n+\frac{n}{2}+\frac{n}{4}+\dots} = \frac{\frac{n}{2}(1+n)}{2n} = \frac{n(n+1)}{2\cdot 2n} = \frac{n+1}{4} = \frac{1}{4}(n+1)$

[ $1+2+3+\dots+n$

AP

$s_n = \frac{n}{2}(a_1+a_n)$

$s_n = \frac{n}{2}(1+n)$ ]

[ $n + \frac{n}{2} + \frac{n}{4} + \dots = n(\underbrace{1+\frac{1}{2}+\frac{1}{4}+\dots}) =$

$= n\cdot 2 = 2n$

[ GŘ: $a_1 = 1$, $q = \frac{1}{2}$

$|q| < 1$ platí

$s = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$ ] ]

⑥ Zjistěte, pro která $x \in R$ jsou řady konvergentní a určete jejich součet

a) $\sum_{n=1}^{\infty} (1-5x)^n = (1-5x)^1 + (1-5x)^2 + (1-5x)^3 + (1-5x)^4 + \dots$

GŘ: $q = \frac{(1-5x)^2}{(1-5x)} = 1-5x$ podm. konverg. $|q| < 1$

$|1-5x| < 1$

$|5x-1| < 1 \quad |:5$

$|x - \frac{1}{5}| < \frac{1}{5}$

$x_0 = \frac{1}{5}$

$0 \quad \frac{1}{5} \quad \frac{2}{5}$

$s = \frac{a_1}{1-q} = \frac{1-5x}{1-(1-5x)} = \underline{\frac{1-5x}{5x}}$ $\Leftarrow$ $x \in (0, \frac{2}{5})$

b) DÚ $\sum_{n=1}^{\infty} \left(\frac{1}{x}\right)^n = \underbrace{\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}} + \dots = x^{-1} + x^{-2} + x^{-3} + \dots$ $x \neq 0$

GŘ: $q = \frac{\frac{1}{x^2}}{\frac{1}{x}} = \frac{x}{x^2} = \frac{1}{x}$ podm. konverg. $|q| < 1$

$|\frac{1}{x}| < 1$

$\underline{s} = \frac{a_1}{1-q} = \frac{\frac{1}{x}}{1-\frac{1}{x}} = \left(\frac{\frac{1}{x}}{\frac{x-1}{x}}\right) = \frac{x}{x(x-1)} = \underline{\frac{1}{x-1}}$

$\frac{1}{|x|} < 1 \quad |\cdot|x|, |x|>0$

$1 < |x|$

$|x| > 1$

$x_0 = 0$

$-1 \quad 0 \quad 1$

$\underbrace{x \in (-\infty, -1) \cup (1, +\infty)}$
podmínka konv. pro $x$

⑦ Řešte v R

a) $\sum_{n=1}^{\infty} \left(\frac{2}{x}\right)^{n-1} = \frac{4x-3}{3x-4}$ $O = R$, $D' = R - \{0, \frac{4}{3}\}$ $\left[\begin{array}{l}3x-4 \neq 0 \\ x \neq \frac{4}{3}\end{array}\right]$

$\underbrace{1 + \frac{2}{x} + \left(\frac{2}{x}\right)^2 + \left(\frac{2}{x}\right)^3 + \dots} = \frac{4x-3}{3x-4}$ $\left[\left(\frac{2}{x}\right)^{1-1} = \left(\frac{2}{x}\right)^0 = 1 \text{ pro } x \neq 0\right]$

GŘ: $q = \frac{2}{x}$, $a_1 = 1 \rightarrow$ podm. konv. $|q| < 1$ pl.

$|\frac{2}{x}| < 1$

$s = \frac{a_1}{1-q} = \frac{1}{1-\frac{2}{x}} = \frac{1}{\frac{x-2}{x}}$

$\frac{2}{|x|} < 1 \quad |\cdot|x|, |x|>0$

$2 < |x|$

$s = \frac{x}{x-2}$

$|x| > 2$

$-2 \quad 0 \quad 2$

- rce má tvar: $\frac{x}{x-2} = \frac{4x-3}{3x-4}$

$x \in (-\infty, -2) \cup (2, +\infty) \cap D'$

$D = (-\infty, -2) \cup (2, +\infty)$

$x(3x-4) = (4x-3)(x-2)$

$3x^2 - 4x = 4x^2 - 8x - 3x + 6$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x_1 = 6 \in D \qquad x_2 = 1 \notin D$

$\underline{K = \{6\}}$

b) $\dfrac{8}{x+10} = \underbrace{1 - \dfrac{3}{x} + \dfrac{9}{x^2} - \dfrac{27}{x^3} + \dots}$

$O = R$
$D' = R - \{0, -10\}$ $\left[ x \neq 0 \quad x+10 \neq 0 \atop x \neq -10 \right]$

GŘ: $a_1 = 1$, $q = -\dfrac{3}{x}$

podm. konv. $|q| < 1$

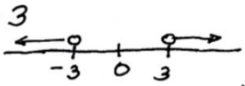
$\left|-\dfrac{3}{x}\right| < 1$

$\dfrac{3}{|x|} < 1$

$3 < |x|$

$|x| > 3$

$[x_0 = 0]$




$D'' = (-\infty, -3) \cup (3, +\infty)$

$s = \dfrac{a_1}{1-q}$

$s = \dfrac{1}{1-\left(-\frac{3}{x}\right)} = \dfrac{1}{1+\frac{3}{x}} = \dfrac{1}{\frac{x+3}{x}} = \dfrac{x}{x+3}$

$D = D' \cap D'' = (-\infty, -10) \cup (-10, -3) \cup (3, +\infty)$

- rce má tvar

$\dfrac{8}{x+10} = \dfrac{x}{x+3}$

$8(x+3) = x(x+10)$

$8x + 24 = x^2 + 10x$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x_1 = -6 \in D$ $\quad x_2 = 4 \in D$

$K = \{-6, 4\}$

c) $\displaystyle\sum_{n=1}^{\infty} (3x)^{n-1} = 10$ $\qquad D' = R$

$(3x)^0 + (3x)^1 + (3x)^2 + \dots = 10$

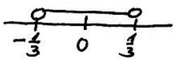
$\underbrace{1 + 3x + 9x^2 + \dots}= 10$

GŘ: $q = \dfrac{9x^2}{3x} = \dfrac{3x}{1} = 3x$ $\qquad$ podm. konv. $|q| < 1$

$|3x| < 1$

$3|x| < 1$

$|x| < \dfrac{1}{3}$




$s = \dfrac{a_1}{1-q} = \dfrac{1}{1-3x}$ $\qquad \Leftarrow \qquad x \in \left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$

rovnice má tvar

$D'' = \left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$

$\dfrac{1}{1-3x} = 10$ $\qquad D = D' \cap D'' = \left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$

$1 = 10(1-3x)$

$1 = 10 - 30x$

$30x = 9$

$x = \dfrac{9}{30} = \dfrac{3}{10} \in D$ $\qquad K = \left\{\dfrac{3}{10}\right\}$

d) $\underbrace{1 - x + x^2 - x^3 + \dots} = \dfrac{\sqrt{2}}{2}$ $\qquad O = R$, $D' = R$

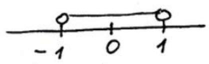
GŘ: $a_1 = 1$, $q = \dfrac{-x}{1} = \dfrac{x^2}{-x} = -x$

podm. konv. $|q| < 1$

$|-x| < 1$

$|x| < 1$

$[x_0 = 0]$




$D'' = (-1, 1)$

$D = D' \cap D'' = R \cap (-1, 1) = (-1, 1)$

$s = \dfrac{a_1}{1-q} = \dfrac{1}{1-(-x)} = \dfrac{1}{1+x}$

- rce má tvar $\qquad D = (-1, 1)$

$\dfrac{1}{1+x} = \dfrac{\sqrt{2}}{2}$

$2 = \sqrt{2}(1+x)$

$2 = \sqrt{2} + \sqrt{2} \cdot x$

$\sqrt{2}\, x = 2 - \sqrt{2}$

$x = \dfrac{2-\sqrt{2}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}(2-\sqrt{2})}{2}$

usměrnit

$x = \dfrac{2\sqrt{2} - 2}{2} = \dfrac{2(\sqrt{2}-1)}{2}$

$x = \sqrt{2} - 1 \in D$ $\qquad [\sqrt{2} - 1 \doteq 1{,}4 - 1 = 0{,}4 \in D]$

$K = \{\sqrt{2} - 1\}$

e) $x + 3x^2 + x^3 + 3x^4 + \dots = \frac{5}{3}$

ř:

rozložíme na 2 řady

$x + x^3 + x^5 + \dots$

GŘ: $a_1 = x \quad q = x^2 \quad |x^2| < 1$

$|x| < 1$

$s_1 = \frac{x}{1-x^2} \quad x \in (-1,1) \quad D'' (-1,1)$

$3x^2 + 3x^4 + 3x^6 + \dots$

$3(\underbrace{x^2 + x^4 + x^6 + \dots}_{})$

GŘ: $a_1 = x^2 \quad q = x^2 \quad |x^2| < 1$

$|x| < 1$

$s_2 = 3\frac{x^2}{1-x^2} \quad x \in (-1,1)$

$D''' (-1,1)$

$D' = R \qquad D = D' \cap D'' \cap D'''$

$D = (-1,1)$

rce má tvar: $s_1 + s_2 = \frac{5}{3}$

$\frac{x}{1-x^2} + \frac{3x^2}{1-x^2} = \frac{5}{3}$

$3x + 9x^2 = 5 - 5x^2$

$14x^2 + 3x - 5 = 0$

$x_{1,2} = \frac{-3 \pm \sqrt{9 + 4.14.5}}{28} = \frac{-3 \pm \sqrt{289}}{28}$

$= \frac{-3 \pm 17}{28} = \left\langle \begin{array}{l} -\frac{20}{28} = -\frac{5}{7} \in D \\ \frac{14}{28} = \frac{1}{2} \in D \end{array} \right.$

$K = \left\{\frac{1}{2}, -\frac{5}{7}\right\}$

⑧ Řešte v R

a) $\log x + \log\sqrt{x} + \log\sqrt[4]{x} + \log\sqrt[8]{x} + \dots = 2$ $\qquad D' = R^+ = (0,+\infty)$ $\quad$ [$x > 0 \wedge x \geq 0$] (pro $\log x$, pro $\sqrt{\ }$)

$\log x^1 + \log x^{\frac{1}{2}} + \log x^{\frac{1}{4}} + \log x^{\frac{1}{8}} + \dots = 2$ $\quad$ [$\log_z x^n = n \log_z x$] $\quad D = D' \cap D'' = (0,+\infty)$

1.zp.

$1\log x + \frac{1}{2}\log x + \frac{1}{4}\log x + \frac{1}{8}\log x + \dots = 2$

$\log x(\underbrace{1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots}_{}) = 2$

GŘ: $a_1 = 1, \quad q = \frac{1}{2} \quad \left(= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}\right)$

podm. konv. $|q| < 1$ platí pro $\forall x \in R$ $\Rightarrow D'' = R$

$s = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

- rce má tvar

$s \cdot \log x = 2$ $\quad$ [$\log_z x = y \Leftrightarrow z^y = x$]

$2\log x = 2$

$\log x = 1$ $\quad$ [$\log_{10} x = 1 \Leftrightarrow 10^1 = x$]

$x = 10 \in D$

$K = \{10\}$

lze i 2.zp.

$\log x + \frac{1}{2}\log x + \frac{1}{4}\log x + \dots = 2$

GŘ: $a_1 = \log x \qquad q = \frac{\frac{1}{2}\log x}{\log x} = \frac{1}{2}$

podm. konv. $|q| < 1$ platí pro $\forall x \in R^+$

$s = \frac{a_1}{1-q} = \frac{\log x}{1-\frac{1}{2}} = \frac{\log x}{\frac{1}{2}}$

$s = 2\log x$

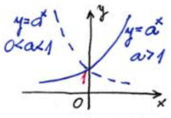
rce má tvar $2\log x = 2 \dots$ viz výše

b) $2^x + 4^x + 8^x + 16^x + \dots = 1$ $\qquad D' = R$

$2^x + (2^2)^x + (2^3)^x + (2^4)^x + \dots = 1$

$\underbrace{2^x + 2^{2x} + 2^{3x} + 2^{4x} + \dots}_{} = 1$

GŘ: $a_1 = 2^x \qquad q = \frac{2^{2x}}{2^x} = \frac{2^x \cdot 2^x}{2^x} = 2^x$




podm. konv. $|q| < 1$

$|2^x| < 1$ $\qquad$ z grafu $\underset{>0}{|2^x|} = 2^x$ !

$2^x < 1$

$2^x < 2^0$

$x < 0$ $\qquad$ exponc. nerc. - pro základ $a = 2 > 1$ ! NEMĚNÍME ZNAK NEROVNOSTI

$D'' = R^- = (-\infty,0)$

$D = D' \cap D'' = (-\infty,0)$

$s = \frac{a_1}{1-q} = \frac{2^x}{1-2^x}$

$D = (-\infty,0)$

- rce má tvar

$\frac{2^x}{1-2^x} = 1$

$2^x = 1 - 2^x$

$2^x + 2^x = 1$

$2 \cdot 2^x = 1$

$2^{x+1} = 1$

$2^{x+1} = 2^0$

$x + 1 = 0$

$x = -1 \in D$

$K = \{-1\}$